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EVOLUTION OF DYNAMICAL FIFTH DIMENSION IN BRANEWORLD COSMOLOGY

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Abstract

Evolutions of the dynamical system in braneworld cosmology are of main issues on astrophysics and cosmology alike. One can think of extra dimension as important contributor of spacetimes evolution and gives the some interesting cosmological solutions. The solutions are in the form of travelling wave-like nature and visualizations of the solutions are given.

Keywords: Braneworld cosmology, extra dimension.

Introduction

There has been recently a lot of activity on the possibility in a three-dimensional world embedded in a higher dimensional space. Modifying the old Kaluza-Klein picture (Bailin, D., A. Love, 1987), where the extra-dimensions must be sufficiently compact, these recent developments are based on the idea that ordinary matter fields could be confined to a three-dimensional world, corresponding to the apparent Universe, while gravity could live in a higher dimensional space (Arkani-Hamed, N., S. Dimopoulos, G. Dvali, 1998). The usual constraints on Kaluza-Klein models could therefore be relaxed (Arkani-Hamed, N., S. Dimopoulos, G. Dvali, 1999), and "large" extra-dimensions would be conceivable, thus leading to a fundamental Planck mass much lower than its apparent three-dimensional value, even as low as the *TeV* scale.

The purpose of the present work is to solve the five-dimensional Einstein's equations for *any type of matter in the brane* with a cosmological constant in the bulk. It can be seen that Einstein's equations admit a first integral, which in particular provides directly the cosmological evolution of the brane. In the following , it will be shown that an additional assumption, namely that the metric along the fifth dimension does not evolve in time, enables us to solve for the whole space-time metric, i.e. to find explicitly the dependence of the metric on the transverse coordinate as well as time. Finally, in the last section, it will solve analytically, for different cases, the new Friedmann equation obtained in the present work and discuss the consequences.

Solving Einstein's equations

Let's present here the general framework. It should be considered five-dimensional space-time metrics of the form

$$ds^2 = \tilde{g}_{AB}dx^A dx^B = g_{\mu\nu}dx^\mu dx^\nu + b^2 dy^2 \tag{1}$$

where y is the coordinate of the fifth dimension. Throughout this article, one will focus our attention on the hypersurface defined by $y = 0$, which one can identify with the world volume of

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the brane that forms our universe. Since one can be interested in cosmological solutions, it can be taken a metric of the form

$$ds^2 = -n^2(\tau, y)d\tau^2 + a^2(\tau, y)\gamma_{ij}dx^i dx^j + b^2(\tau, y)dy^2 \tag{2}$$

Where γ_{ij} is a maximally symmetric 3-dimensional metric (k = −1, 0,1 will parametrize the spatial curvature).

The five-dimensional Einstein equations take the usual form

$$\tilde{G}_{AB} = \tilde{R}_{AB} - \frac{1}{2}\tilde{R}_{\tilde{g}_{AB}} = \kappa^2 \tilde{T}_{AB} \tag{3}$$

where $\tilde{R}_{AB}$ is the five-dimensional Ricci tensor and $\tilde{R} = \tilde{g}^{AB}\tilde{R}_{AB}$ the scalar curvature and the constant κ is related to the five-dimensional Newton's constant $G_{(5)}$ and the five-dimensional reduced Planck mass $M_{(5)}$, by the relations

$$\kappa^2 = 8\pi G_{(5)} = M_{(5)}^{-3} \tag{4}$$

Inserting the ansatz (2) for the metric, the non-vanishing components of the Einstein tensor $\tilde{G}_{AB}$ are found to be

$$\tilde{G}_{00} = 3\left\{\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) - \frac{n^2}{b^2}\left(\frac{a''}{a} + \frac{a'}{a}\left(\frac{a'}{a} - \frac{b'}{b}\right)\right) + k\frac{n^2}{a^2}\right\}, \tag{5}$$

$$\tilde{G}_{ij} = \frac{a^2}{b^2}\gamma_{ij}\left\{\frac{a'}{a}\left(\frac{a'}{a} + 2\frac{n'}{n}\right) - \frac{b'}{b}\left(\frac{n'}{n} + 2\frac{a'}{a}\right) + 2\frac{a''}{a} + \frac{n''}{n}\right\}$$

$$+\frac{a^2}{n^2}\gamma_{ij}\left\{\frac{\dot{a}}{a}\left(-\frac{\dot{a}}{a} + 2\frac{\dot{n}}{n}\right) - 2\frac{\ddot{a}}{a} + \frac{\dot{b}}{b}\left(-2\frac{\dot{a}}{a} + \frac{\dot{n}}{n}\right) - \frac{\ddot{b}}{b}\right\} - k\gamma_{ij} \tag{6}$$

$$\tilde{G}_{05} = 3\left(\frac{n'}{n}\frac{\dot{a}}{a} + \frac{a'}{a}\frac{\dot{b}}{b} - \frac{\dot{a}'}{a}\right) \tag{7}$$

$$\tilde{G}_{55} = 3\left\{\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - \frac{b^2}{a^2}\left(\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\right) + \frac{\ddot{a}}{a}\right) - k\frac{b^2}{a^2}\right\}. \tag{8}$$

In the above expressions, a prime stand for a derivative with respect to y, and a dot for a derivative with respect to τ.

The stress-energy-momentum tensor can be decomposed into two parts,

$$\tilde{T}_B^A = \breve{T}_B^A\Big|_{bulk} + T_B^A\Big|_{brane} \tag{9}$$

where $\breve{T}_B^A\Big|_{bulk}$ is the energy momentum tensor of the bulk matter, which will be assumed in the present work to be of the form

$$\breve{T}_B^A\Big|_{bulk} = diag(-\rho_B, P_B, P_B, P_B, P_T), \tag{10}$$

where the energy density ρ_B and pressures P_B and P_T are independent of the coordinate *y*. Later, one will be specially interested in the case of a cosmological constant for which $-\rho_B = P_B = P_T$.

The second term $T_B^A\big|_{brane}$ corresponds to the matter content in the brane ($y = 0$). Since it can be considered here only strictly homogeneous and isotropic geometries inside the brane, the latter can be expressed quite generally in the form

$$T_B^A\big|_{brane} = \frac{\delta(y)}{b}\, diag(-\rho_b, P_b, P_b, P_b, 0), \tag{11}$$

where the energy density ρ_b and pressure p_b are independent of the position inside the brane, i.e. are functions only of time adding some other brane sources with similar energy momentum tensor.

The assumption that $\tilde{T}_{05} = 0$, which physically means that there is no flow of matter along the fifth dimension, implies that $\tilde{G}_{05}$ vanishes. It then turns out, remarkably, that the components (0, 0) and (5, 5) of Einstein's equations, in the bulk, can be rewritten in the simple form

$$F' = \frac{2a'a^3}{3} k^2 \breve{T}_0^{\;0}, \tag{12}$$

$$\dot{F} = \frac{2\dot{a}a^3}{3} k^2 \breve{T}_5^{\;5}, \tag{13}$$

where *F* is a function of τ and *y* defined by

$$F(\tau, y) = \frac{(a'a)^2}{b^2} - \frac{(\dot{a}a)^2}{n^2} - ka^2. \tag{14}$$

Since $\breve{T}_0^0 = -\rho_B$ is here independent of *y*, one can integrate Equation (12), which gives

$$F + \frac{k^2}{6} a^4 \rho_B + C = 0, \tag{15}$$

where *C* is a constant of integration which a priori depends on time. Assuming in addition that, $\breve{T}_0^0 = \breve{T}_5^5$ one finds using the time derivative of Equation (12) and the *y*-derivative of Equation (13) that is constant in time. This also implies that *C* is constant in time. In order to deal with the last component of Einstein's equations, it is convenient to use the Bianchi identity

$$\nabla_A \tilde{G}^{A0} = 0, \tag{16}$$

which can be rewritten

$$\partial_\tau\left(\frac{F'}{a'}\right) = \frac{2}{3}\dot{a}a^2 \gamma_j^i \tilde{G}_i^j. \tag{17}$$

One finds that this equation is identically satisfied if $-\rho_B = P_B$. Hence, when the bulk source is a cosmological constant, any set of functions *a*, *n*, and *b* satisfying Equation (15) or, more explicitly,

$$\left(\frac{\dot{a}}{na}\right)^2 = \frac{1}{6}k^2\rho_B + \left(\frac{a'}{ba}\right)^2 - \frac{k}{a^2} + \frac{C}{a^4} \tag{18}$$

together with $\tilde{G}_{05} = 0$, will be solution of all Einstein's equations Equation (3), locally in the bulk.

The brane can then be taken into account by using the junction conditions, which simply relate the jumps of the derivative of the metric across the brane to the stress-energy tensor inside the brane. The relevant expressions are

$$\frac{[a']}{a_0 b_0} = -\frac{k^2}{3}\rho_b, \tag{19}$$

$$\frac{[n']}{n_0 b_0} = \frac{k^2}{3}(3\rho_b + 2\rho_b), \tag{20}$$

where the subscript 0 for *a; b; n* means that these functions are taken in $y = 0$, and $[Q] = Q(0^+) - Q(0^-)$ denotes the jump of the function Q across $y = 0$.

Assuming the symmetry $y \leftrightarrow -y$ for simplicity, the junction condition in Equation (19) can be used to compute a' on the two sides of the brane, and by continuity when $y \to 0$, (18) will yield the generalized (first) Friedmann equation

$$\frac{\dot{a}_0^2}{a_0^2} = \frac{k^2}{6}\rho_B + \frac{k^4}{36}\rho_b^2 + \frac{C}{a_0^4} - \frac{k}{a_0^2}. \tag{21}$$

The salient features of this equation are that, first, the bulk energy density enters linearly, second, the brane energy density enters quadratically, and finally the cosmological evolution depends on a free parameter C (related to the choice of initial conditions in the whole space-time), whose influence corresponds to an effective radiation term.

Explicit dependence on the fifth dimension for a stabilized bulk

In this section, it will be shown that, with the help of an additional assumption, namely that the fifth dimension is static, in the sense that

$$\dot{b} = 0 \tag{22}$$

it is then possible to solve the full space-time metric, i.e. to determine the explicit dependence of the metric on the coordinate *y*. The restriction Equation (22) allows to go to the gauge

$$b = 1 \tag{23}$$

It then follows immediately from equation $\tilde{G}_{05} = 0$, that ***n*** can be expressed in terms of ***a*** according to the relation

$$\frac{\dot{a}}{n} = \alpha(t), \tag{24}$$

where α is a function that depends only on time (and not on *y*). Inserting this into Equation (12) yields the following differential equation

$$\alpha^2 + k - (aa')' = \frac{k^2}{3}\rho_B a^2 \tag{25}$$

which is valid everywhere in the bulk (but not in the brane) on the two sides of the brane separately. It can be integrated in *y*, yielding

$$a^2 = A\cosh(\mu y) + B\sinh(\mu y) + C \tag{26}$$

with

$$\mu = \sqrt{-\frac{2k^2}{3}\rho_B}, \tag{27}$$

in the case of $\rho_B < 0,$ or

$$a^2 = A\cos(\mu y) + B\sin(\mu y) + C \tag{28}$$

with

$$\mu = \sqrt{\frac{2k^2}{3}\rho_B}, \tag{29}$$

in the case of $\rho_B > 0,$ or finally

$$a^2 = (\alpha^2 + k)y^2 + Dy + E, \tag{30}$$

for ρ_B = 0. In the following, it should focus on the first case $\rho_B < 0,$ but all the equations will apply as well to the case $\rho_B > 0,$ up to the transformation $\mu \to i\mu, B \to iB.$

The coefficients *A*, *B*, *C*, *D*, *E* are functions of time, and *C* is expressible in terms of α as

$$C = 3\frac{\alpha^2 + k}{k^2\rho_B}, \tag{31}$$

and the others can be determined by the junction conditions. The symmetry $y \leftrightarrow -y$ imposes the relations $A_+ = A_- \equiv \overline{A},\ \ B_+ = -B_- \equiv \overline{B}$ between the coefficients on the two sides of the brane. Using Equation (19) and Equation (20), one then finds

$$\frac{\mu\overline{B}}{\overline{A}+C} = -\frac{k^2}{3}\rho_b,\ \ \frac{2\mu\dot{\overline{B}}}{\dot{\overline{A}}+\dot{C}} = k^2\left(p_b + \frac{1}{3}\rho_b\right). \tag{32}$$

Note that one can check explicitly energy conservation in the brane from these relations, i.e.

$$\dot{\rho}_b + 3\frac{\dot{a}_0}{a_0}(\rho_b + p_b) = 0. \tag{33}$$

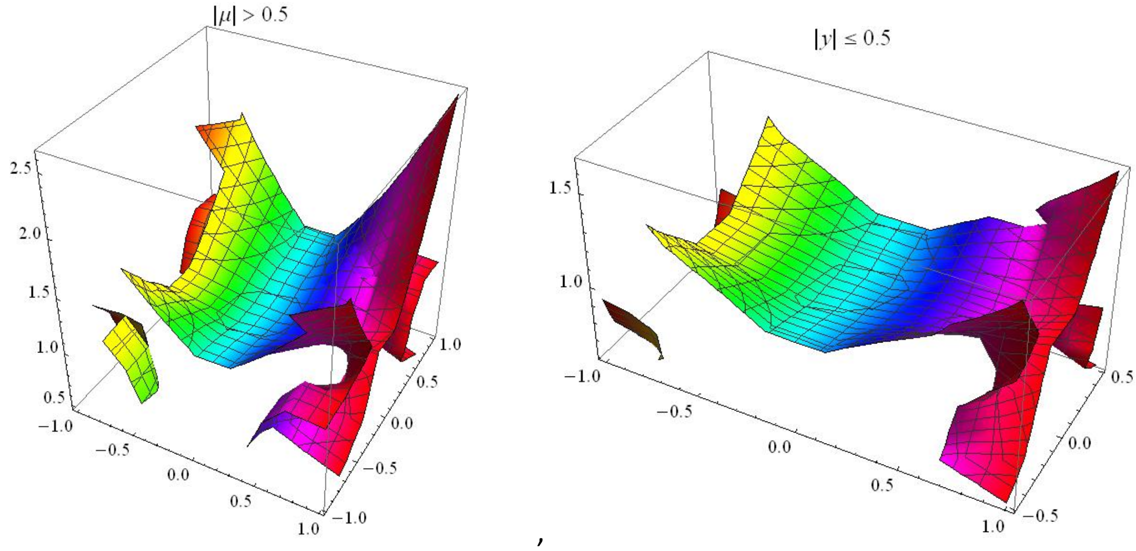

Figure 1 List surface 3D plot of ***a*** in terms of ***μ*** and ***y***

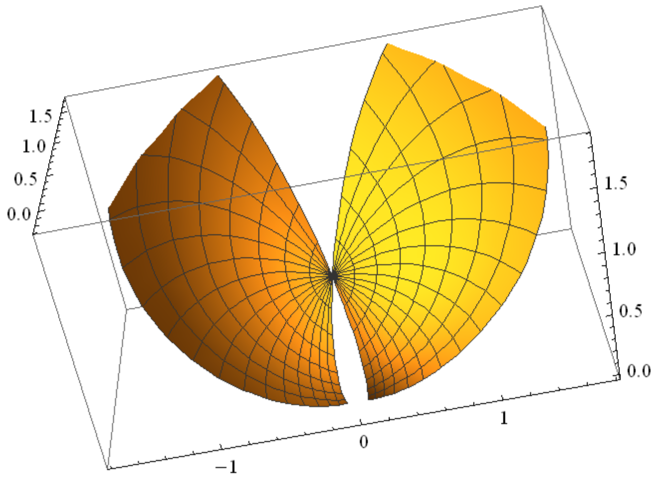

Figure 2 Spherical 3D plot of **a** in terms of **μ** and **y**

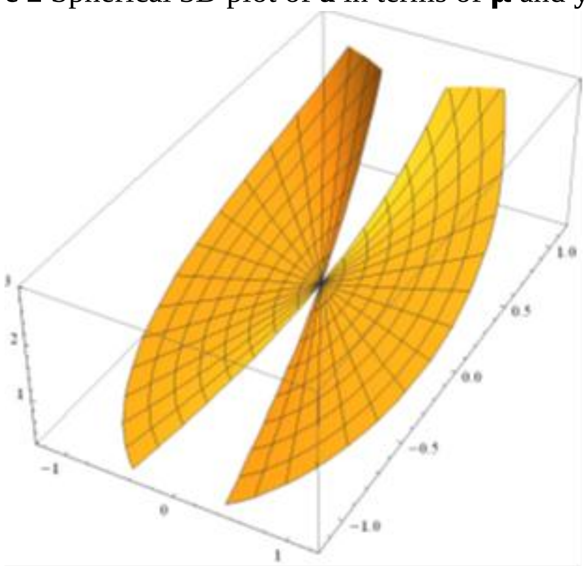

Figure 3 Revolution 3D plot of **a** in terms of **μ** and **y**

Conclusions

It is concluded that the brane structure is of main importance in higher dimension spacetimes. First, it has been shown that one can obtain a first integral of Einstein's equations, which provides, on the brane, a relation analogous to the (first) Friedmann equation and which depends only on the geometry and matter content of the brane, except for a constant parameter. Second, when $\dot{b} = 0$, one can extend explicitly the solution found on the brane to the whole space-time. The travelling waves solution has been visualized and formal wave-like patterns are retained.

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